Ex 13

a) Determiner $Y = \sin(\pi X)$ si $X \rightsquigarrow U[-1;1]$.

$F_Y(y) = P(Y \leq y) = P(\sin \pi X \leq y)$

$= \begin{cases} 1 \text{ si } y \geq 1 \\ 0 \text{ si } y \leq -1 \end{cases}$

$P(X \in [-1; \frac{1}{\pi} \arcsin y] \cup [1 - \frac{1}{\pi} \arcsin y, 1])$ si $y \in]-1;1[$.

$F_Y(y) = \frac{2}{\pi} \arcsin y$.

$f_Y(y) = \frac{dF_Y}{dy} = \begin{cases} \frac{2}{\pi} \frac{1}{\sqrt{1-y^2}} \text{ si } y \in]-1;1[\\ 0 \text{ sinon} \end{cases}$

② $F_Z(z) = P(Z \leq z) = P(\tan X \leq z) = P(X \leq \arctan z)$

$= F_X(\arctan z)$

$f_Z(z) = \frac{dF_Z}{dz} = \frac{1}{1+z^2} \cdot f_X(\arctan z)$.

$= \frac{1}{\pi} \cdot \frac{1}{1+z^2}$ densité $\rightsquigarrow$ Cauchy